On the Long Distance Contribution to the $B_s \to \gamma\gamma$ Decay in the Effective Lagrangian Approach

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Abstract

We re-estimate the decay branching ratio of $B_s \to 2\gamma$ through $D_s^+ D_s^-$ and $D_s^{*+} D_s^{*-}$ intermediate states, in the effective Lagrangian approach. We find that the branching ratio does not exceed a few times 10^{-7}, contrary to the result recently claimed in the literature.

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The $B_s \to \gamma\gamma$ decay is an intereting process which has been investigated by several groups [1–3] within the context of pQCD. The branching ratio was found to be $\mathcal{BR}(B_s \to \gamma\gamma) \sim 3.8 \times 10^{-7}$ for $m_t \simeq 175GeV$, to be compared with the present experimental upper limit $\mathcal{BR}(B_s \to \gamma\gamma) < 1.48 \times 10^{-4}$ [4]. It was also pointed out that the branching ratio can be substantially enhanced in some extensions of the standard model such as a generic 2-Higgs doublet model [5]. Provide that the short distance contribution is not large [6,5,7], one may then hope the observation of such channel in future experiments as a signal of physics beyond standard model. However, as have been emphasized by Choudhury and Ellis [8], a careful analysis to the long distance effects has to be made before one could use this channel as a probe to the physics beyond the standard model. Choudhury and Ellis have considered the contributions due to intermediate D_s and D_s^* states via the diagrams including loops of D_s mesons alone, loops of D_s^* mesons alone (see fig. 1), and diagrams involving radiative $D_s^* \to D_s + \gamma$ transitions. Based upon an estimation on the absorptive part of the amplitude they find that the contribution due to D_s alone is,

$$Br(B_s \to 2D_s \to \gamma\gamma) \sim 2.9 \times 10^{-8} \ , \tag{1}$$

which is insignificant as comparing with the pQCD short distance contribution. Diagrams involving the $D_s^* \to D_s\gamma$ transition are again found to be small. However they find that the contribution due to D_s^* is much larger,

$$Br(B_s \to 2D_s^* \to \gamma\gamma) \sim 6.5 \times 10^{-6} \ , \tag{2}$$

which, if correct, is of great phenomenological interets as it will be easily seen in future hadron colliders. Because of its importance it is worthwhile to give a careful re-analysis to such a process.

FIGURES

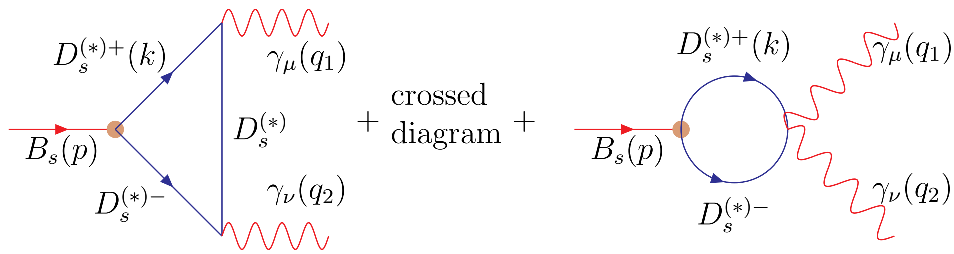

FIG. 1. *One-loop contribution to the $B_s \to \gamma\gamma$ amplitude due to D_s (D_s^*) alone.*

We in the following use the same method as was adopted in ref. [8], i.e., a diagramatic analysis of fig. 1 based on an effective Lagrangian approach. Beside those having been done in ref. [8] we also estimate the real part contribution of the rescattering amplitude using dispersion relations. The Feynman rules we use for the calculation are derived from the dimension 4, $U_{em}(1)$ gauge invariant effective Lagrangian which are bilinear in D_s fields:

$$\mathcal{L}_{eff} = \nabla_\mu D^+ \nabla^\mu D^- - \frac{1}{2}\left(\nabla_\mu D_\nu^- - \nabla_\nu D_\mu^-\right)^+ \left(\nabla^\mu D^{-\nu} - \nabla^\nu D^{-\mu}\right) + ie\phi_V D_\mu^+ F^{\mu\nu} D_\nu^- \ , \tag{3}$$
$$\nabla_\mu = \partial_\mu - ieA_\mu \ ,$$

where ϕ_V is an unknown parameter not constrained by gauge invariance. We have verified that the choice made in ref. [8] corresponds to $\phi_V = 0$. In the case of elementary $W^+W^-\gamma$ interaction $\phi_V = 1$ and in the case of effective $\rho^+\rho^-\gamma$ or $K^{*+}K^{*-}\gamma$ interactions the parameter ϕ_V can be estimated from the extended Nambu–Jona-Lasinio model and is found to be slightly less than 1 [9] (Our definition of ϕ_V is two times larger than Ref. [9]). Since we did not find convincing reason to support taking $\phi_V = 0$, we treat ϕ_V as a free parameter ranging from 0 to 1.

The absorptive part contribution of fig. 1 can be written as,

$$\mathrm{Im}A = \Sigma A_0^{\lambda_1\lambda_2} \beta T_s^{\lambda_1\lambda_2,\lambda_3\lambda_4} \ , \tag{4}$$

where $\beta = \sqrt{1 - 4m^2/s}$ is the kinematic factor, the sum is over the helicity indices λ_1, λ_2 of the intermediate states (for D^{*+} and D^{*-} scattering) and λ_3, λ_4 are the helicity indices of the outgoing photons; (λ_1, λ_2) can take $(+,+)$, $(-,-)$ and $(0,0)$, and (λ_3, λ_4) can take $(+,+)$ and $(-,-)$. In above A_0 is the decay vertex obtained from factorization method,

$$A_0 = \langle D_s^+(k)\ D_s^-(q)|H_{wk}|B_s(p)\rangle = -igf\ \left[(p^2 - k^2)f^+(q^2) + q^2 f^-(q^2)\right] \ , \tag{5}$$

for $B_s \to D_s^+ D_s^-$, and,

$$\begin{aligned} A_0 &= \langle D_s^{*+}(k)\ D_s^{*-}(q)|H_{wk}|B_s(p)\rangle \\ &= gf_*\epsilon_+^{*\mu}\epsilon_-^{*\nu}\left[\mathcal{V}(q^2)\epsilon^{\nu\mu\alpha\beta}p_\alpha k_\beta + i\{\mathcal{A}_1(q^2)p^2 g^{\mu\nu} - 2\mathcal{A}_2(q^2)p^\mu p^\nu\}\right] \ , \end{aligned} \tag{6}$$

for $B_s \to D_s^{*+} D_s^{*-}$. In above equations, $g = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^*$, $f, f^* \simeq 200MeV$, $f_\pm, \mathcal{V} \simeq 0.6$ and $\mathcal{A}_1, \mathcal{A}_2 \simeq 0.25$, according to Ref. [8]. The matrix element for the decay $B_s \to \gamma(q_{1\mu})\gamma(q_{2\nu})$ are parametrized as,

$$
\begin{aligned}
A &= \alpha g \left(R_1 S_{\mu\nu} + i R_2 P_{\mu\nu}\right) \ , \\
S_{\mu\nu} &\equiv q_{1\nu} q_{2\mu} - q_1 \cdot q_2 g_{\mu\nu} \ , \\
P_{\mu\nu} &\equiv \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \ ,
\end{aligned} \tag{7}
$$

where R_1 and R_2 are the yet-to-be-determined hadronic matrix elements, and the partial width is then given by,

$$
\Gamma(B_s \to \gamma\gamma) = \frac{(\alpha g)^2}{64\pi} m_B^3 \left(|R_1|^2 + |R_2|^2\right) \ . \tag{8}
$$

Since $\Gamma(B_s) \simeq 5 \times 10^{-10} MeV$, we have, for $|V_{cb}| = 0.04$,

$$
Br(B_s \to \gamma\gamma) \simeq 10^{-7} \ \left(|\frac{R_1}{100MeV}|^2 + |\frac{R_2}{100MeV}|^2 \right) \ . \tag{9}
$$

In re-calculating the processes, for the $D_s^+ D_s^-$ intermediate state we confirm the result of Ref. [8], Eq. (1). For the $D_s^{*+} D_s^{*-}$ intermediate state, we have,

$$
\begin{aligned}
\mathrm{Im} R_1 = & \frac{f_*}{8\tilde{m}^2} \mathcal{A}_1 \{ [(1-4\tilde{m}) + \phi_V(4\tilde{m}) - \phi_V^2] \lambda_{22} \\
& + \tilde{m}[(1 - 12\tilde{m} + 48\tilde{m}^2) + \phi_V(-2 - 8\tilde{m}) + \phi_V^2(1 - 4\tilde{m})] \log(\frac{1-\beta}{1+\beta}) \} \\
& - \frac{f_*}{8\tilde{m}^2} \mathcal{A}_2 \{ [(1-5\tilde{m}) + \phi_V(2\tilde{m}) + \phi_V^2(-1 + 3\tilde{m})] \lambda_{22} \\
& + \tilde{m}[(1 - 10\tilde{m} + 32\tilde{m}^2) + \phi_V(-2 + 4\tilde{m}) + \phi_V^2(1 - 2\tilde{m})] \log(\frac{1+\beta}{1-\beta}) \} \ , \\
\mathrm{Im} R_2 = & \frac{f_* \mathcal{V}}{16\tilde{m}} \{ [(-1 + 12\tilde{m}) + \phi_V(2 - 8\tilde{m}) + \phi_V^2(-1 - 4\tilde{m})] \lambda_{22} \\
& + [(4\tilde{m} - 32\tilde{m}^2) + \phi_V(8\tilde{m} - 32\tilde{m}^2) + \phi_V^2(4\tilde{m})] \log(\frac{1+\beta}{1-\beta}) \} \ .
\end{aligned} \tag{10}
$$

The calculation made in ref. [8] corresponding to taking $\phi_V = 0$ in above equations. We find a disagreement on the first term inside the square bracket in the coefficient of $\mathcal{A}_2$. As a consequence, the CP conserving branching ratio becomes much smaller, $\mathcal{BR}(B_s \to 2\gamma) \simeq 0.85 \times 10^{-8}$. Meanwhile we reconfirm the numerical result on the CP violating branching ratio of ref. [8]. The maximal value of $\mathcal{BR}(B_s \to 2\gamma)$ comes from the case $\phi_V = 1$. In this situation, our calculation is equivalent to taking the $W^+W^-\gamma$ vertics (propagators are in the unitary gauge). Hence we list some intermediate steps in the calculation here since it may be useful elsewhere. The s partial–wave projection of the $2 \to 2$ scattering amplitudes are found to be,

$$
\beta T_s(D_s^+ D_s^- \to 2\gamma) = \frac{\alpha_e}{2s}(1-\beta^2) \ln(\frac{1+\beta}{1-\beta}) \ , \tag{11}
$$

and,

$$
\beta T_s^{++}(D_s^{*+} D_s^{*-} \to 2\gamma) = \frac{\alpha_e}{2s}(1+\beta)^2 \ln\left(\frac{1+\beta}{1-\beta}\right) \ ,
$$

$$\beta T_s^{--}(D_s^{*+}D_s^{*-} \to 2\gamma) = \frac{\alpha_e}{2s}(1-\beta)^2 \ln\left(\frac{1+\beta}{1-\beta}\right) \ ,$$
$$\beta T_s^{00}(D_s^{*+}D_s^{*-} \to 2\gamma) = \frac{\alpha_e}{8m_{D^*}^2}(1-\beta^2)^2 \ln\left(\frac{1+\beta}{1-\beta}\right) \ . \quad (12)$$

In above the gauge invariant factor $\epsilon_1 \cdot \epsilon_2 k_1 \cdot k_2 - \epsilon_1 \cdot k_2 \epsilon_2 \cdot k_1$ $(= s/2)$ has been taken away, and the photon helicity is taken as (+,+). Once again we find that the $D_s^{*+}D_s^{*-}$ intermediate state contribution to the decay branching ratio of $B_s \to \gamma\gamma$ quite small, though it is comparable in magnitude to the short distance contributions,

$$\mathcal{BR}_{CP-even}(B_s \to 2D_s^* \to 2\gamma) \sim 1.49 \times 10^{-7} \ , \ \mathcal{BR}_{CP-odd}(B_s \to 2D_s^* \to 2\gamma) \sim 0.44 \times 10^{-7} \ . \quad (13)$$

From the above results we find that the decay branching ratio of $B_s \to \gamma\gamma$ will not exceed a few times 10^{-7} provide that the real part contribution does not change the above estimtate in order of magnitude. It is natural to expect that the real part contribution is comparable in order of magnitude to the absorptive part. Therefore, if the future experiments reveal a large branching ratio of $B_s \to 2\gamma$ upto a few times 10^{-6} it is very likely that it is a signal of new physics.

Now we discuss the real part contribution of the process depicted in fig. 1. The real part contribution can be calculated using dispersion relation. When performing the dispersion integral it is necessary to extract the $\epsilon_1 \cdot \epsilon_2 k_1 \cdot k_2 - \epsilon_1 \cdot k_2 \epsilon_2 \cdot k_1$ part out of the dispersive integral [10] since it is the remaining part to be considered as the form-factor.[1] We get,

$$\mathbf{A} = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{\mathrm{Im}\mathbf{A}}{s'-s-i\epsilon} ds' \ . \quad (14)$$

The D_s loop in fig. 1 contains no ultra-violet divergence and it is straightforward to evaluate Eq. (14), we find,

$$\mathcal{BR}(B_s \to 2D_s \to \gamma\gamma) \sim 4.2 \times 10^{-8} \ . \quad (15)$$

For the D_s^* contribution the diagram fig. 1 is ultra-violet divergent and needs to be renormalized in the standard perturbation calculation. In the dispersive approach the divergence manifests itself by the fact that the dispersive integral in Eq. (14) needs one subtraction. The divergence can also be rescued by introducing form-factors in the Feynman vertex to regulate the high energy behaviour of the Feynman diagram. However, it is equivalent in principle to regulate Eq. (14) by using an un-subtracted dispersion relation with truncated integrand,

$$\mathbf{A} = \frac{1}{\pi}\int_{4m^2}^{\Lambda^2} \frac{\mathrm{Im}\mathbf{A}}{s'-s-i\epsilon} ds' \ . \quad (16)$$

Of course, the ultra-violet divergence emerged from the effective Lagrangian approach soley indicates that the effective Lagrangian approach gives a bad high energy behaviour to the

[1]For earlier referrences on the usage of dispersion relation under such a situation, see ref. [10,11].

rescattering amplitude. For example, the $T_s^{++,++}$ amplitude of Eq. (12), behaves as $\sim \frac{1}{s}\ln(s)$ as $s \to \infty$ and does not satisfy Eq. (14). However, from Regge theory we know that, at high energies, the s–wave scattering amplitudes behave as $T_s(s) \sim s^{\alpha_{D_s}-1}$ and $T_s(s) \sim s^{\alpha_{D_s^*}-1}$, for the D_s and D_s^* exchanges, respectively. The intercept parameters of the Regge trajectory are $\alpha_{D_s} \simeq -1.5$ and $\alpha_{D_s{}^*} \simeq -1$. It is clear that the dispersion relation, Eq. (14), is convergent. The rapid fall-off behaviour of the Regge amplitude with respect to s implies that the magnitude of the real part contribution obtained in the effective Lagrangian approach and via Eq. (16) be well overestimated. In fact, Regge behaviour should already dominate at $s = M_{B_s}^2$ [12]. So the result obtained from Eq. (16) may at best be interpreted as an upper bound. Since the high energy contribution can not be important to the dispersive integral in Eq. (14), we take $\Lambda \sim 2M_{B_s}^2$ as an educative estimation. We find that the inclusion of the real part contribution does not alter the order of magnitude of the absorptive contribution. For example, when taking $\phi_V = 1$, we have,

$$\mathcal{BR}(B_s \to 2D_s^* \to 2\gamma) \sim 3.7 \times 10^{-7} \ , \quad \Lambda = 2M_{B_s}^2 \ , \tag{17}$$

to be compared with Eq. (13).

To conclude, we have re-evaluated the $B_s \to 2\gamma$ decay process through $D_s^+ D_s^-$ and $D_s^{*+} D_s^{*-}$ intermediate states' rescatterings. We have estimated the absorptive part contribution to the decay branching ratio in the effective Lagrangian approach, and also estimated the real part contribution through a cut-off regulated dispersion relation. We argue that the decay branching ratio will not exceed a few times 10^{-7}. Therefore if the furture experiments reveal a large branching ratio of a few times 10^{-6} it must come from new physics effects.

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